# Importance of the Near Wake in Drag Prediction of Bodies of Revolution

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## Introduction

IN the conventional method of drag prediction for a body of revolution at zero incidence, the drag coefficient is calculated, in the absence of separation, first by determining the development of the boundary layer up to the tail and then making use of the formula of either Young[1] or Granville.[2] These formulas are derived on the assumption that the development of the wake downstream of the tail takes place according to the momentum-integral equation,

$$(\mathrm{d}\Theta/\mathrm{d}x)+(H+2)\,(\Theta/U_e)\,(\mathrm{d}U_e/\mathrm{d}x)=0 \tag{1}$$

where $H=\Delta^*/\Theta$ is the shape parameter, $\Delta^*$ is the mass-deficit area, $\Theta$ is the momentum-deficit area, $U_e$ is the velocity at the edge of the wake, and $x$ is the downstream distance along the axis of the wake. The integral areas are defined by

$$\Delta^*=\int_0^\delta \left(1-\frac{U}{U_e}\right)r\mathrm{d}r,\qquad \Theta=\int_0^\delta \frac{U}{U_e}\left(1-\frac{U}{U_e}\right)r\mathrm{d}r \tag{2}$$

where $r$ is the radial distance from the axis of the body (or wake), $\delta$ is the wake thickness, and $U(r,x)$ is the axial velocity distribution across the wake.

The integration of Eq. (1) from the tail of the body (subscript $t$) to far downstream of the body (subscript $\infty$), where $U_e=U_\infty$ and $H=1$, leads to

$$\Theta_\infty=\Theta_t\left(\frac{U_e}{U_\infty}\right)_t^{H_t+2}\exp\left\{\int_1^{H_t}\ln\frac{U_\infty}{U_e}\mathrm{d}H\right\} \tag{3}$$

Young[1] assumed a linear variation of $\ln(U_\infty/U_e)$ with $H$, whereas Granville[2] suggested the use of

$$\ln(U_\infty/U_e)=\ln(U_\infty/U_e)_t[(H-1)/H_t-1)]^q \tag{4}$$

with $q=7$. Note that Young's assumption corresponds to $q=1$. The integral in Eq. (3) can be evaluated using Eq. (4) to obtain the drag coefficient $C_D$ in the form

$$C_D\equiv\frac{D}{\frac{1}{2}\rho U_\infty^2 S}=\frac{4\pi\Theta_\infty}{S}=\frac{4\pi}{S}\Theta_t\left(\frac{U_e}{U_\infty}\right)_t^{H_t+2}\left(\frac{U_e}{U_\infty}\right)_t^{(1-H_t)/(1+q)} \tag{5}$$

Here, $D$ is the drag, $S$ is a representative area of the body, and $\rho$ is the mass density of the fluid.

Although the formulas of Young ($q=1$) and Granville ($q=7$) have been used with remarkable success for a long time (see, for example, Nakayama and Patel[3]), large discrepancies have been observed recently between the values of the drag coefficient predicted by these formulas and those deduced from measurements in the wake. The purpose of this Note is to explore the reasons for these differences.

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## Analysis of Near-Wake Data

In a recent paper, Patel, et al.[4] reported measurements in the thick boundary layer over the tail of a modified spheroid. Subsequently, measurements of mean velocity distributions[5] also were made in the near wake of that body. Similar experiments presently are being performed using a low-drag body of revolution (the F-57 body of Parsons and Goodson[6]). In both cases the boundary layer remains attached up to the tail and becomes quite thick. The absence of separation enabled the detailed measurements in the near wake. Since the boundary-layer characteristics were measured at the tail, it also was possible to apply Eq. (5), with $q=1$ and $q=7$, directly to determine the drag coefficient. Comparison of the drag coefficients predicted in this manner with those determined from the wake measurements well downstream indicated, however, that Eq. (5), with both values of $q$, overestimated the drag coefficient by about 19% in the case of the low-drag body and by about 30% for the modified spheroid. This appeared somewhat surprising especially in view of the success achieved earlier[3] in dealing with bodies of more conventional shapes. The available near-wake data were therefore analyzed to elucidate the origin of the observed differences.

Figure 1 shows the variation of the momentum-deficit area, $\Theta/L^2$ with downstream distance $x/L$ in the wake of the two bodies ($x/L=1$ at the tail). Also shown in the figure are the values of $\Theta_\infty/L^2$ computed from Eq. (5) using the known information at the tail as well as the computed value of $\Theta_\infty/L^2$ for the modified spheroid obtained by an iterative (see below) method.[5,7,8] Here, $L$ is the axial length of the body. Figure 1 shows immediately that the drag coefficient predicted by Eq. (5) will be substantially higher than the actual value, since the predicted values of $\Theta_\infty$ are larger than asymptotic values observed by experiment.

There are two possible reasons for the discrepancies just noted. The first stems from the assumption concerning the variation of the velocity outside the near wake, namely $U_e/U_\infty$. Figure 2 shows a comparison between the assumptions of Young and Granville and the experimental data. Although there are large differences in this respect, it is interesting to note that the assumption concerning the variation of $U_e/U_\infty$ with $H$ is not crucial in the determination of $\Theta_\infty$ since its integral is the argument of the expotential in Eq. (3) and the exponential term is very close to unity with either assumption. This is well demonstrated by the observation that the values of $\Theta_\infty$ predicted by using the formulas of Young and Granville are not very different (Fig. 1) although their assumptions are vastly different (Fig. 2). The second, and more important, reason for the failure of the drag-prediction formulas lies in the basic assumption that the development of the near wake obeys the usual momentum-integral equation, Eq. (1). Recall that this equation is derived under the assumption that the pressure is substantially constant across the wake. The experiments of Patel, et al.[4] and those being made on the low-drag body of revolution suggest, however, that the rapid growth of the boundary layer over the tail of the body results in substantial variation of pressure across the boundary layer and the near wake. The momentum-integral equation for the near wake therefore takes the form[5]

$$\frac{\mathrm{d}\Theta}{\mathrm{d}x}+(2+H)\frac{\Theta}{U_e}\frac{\mathrm{d}U_e}{\mathrm{d}x}=\frac{1}{U_e^2}\int_0^\delta r\frac{\partial}{\partial x}\left\{\frac{p-p_e}{\rho}-\frac{V_e^2}{\rho}\right\}\mathrm{d}r\equiv I_p \tag{6}$$

where $p$ is the pressure in the wake, and $p_e$ and $V_e$ are, respectively, the pressure and the radial velocity at the edge of the wake. The contribution of the Reynolds stresses to this equation has been neglected on the basis of experimental evidence. However, since the experiments indicate that the pressure-variation term $I_p$ is negative in the near wake, the values of $\Theta$ predicted by ignoring it are larger. This results in

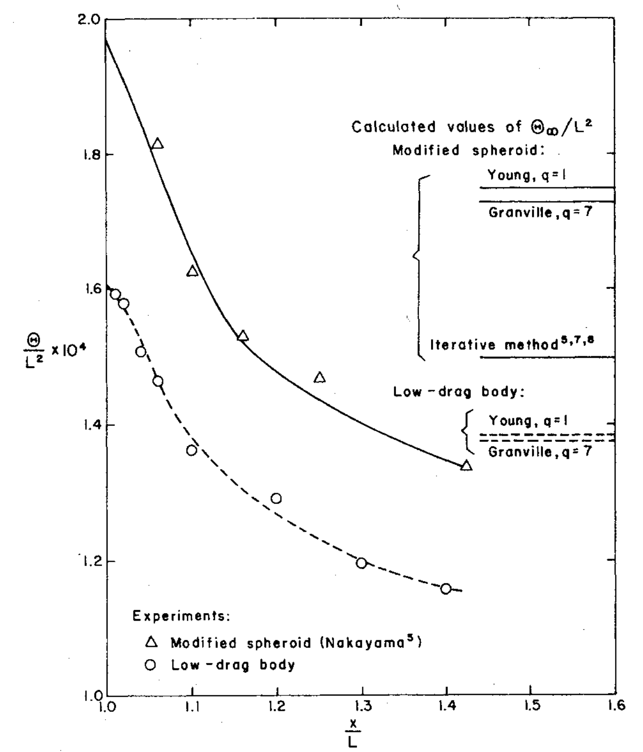


Fig. 1 Variation of $\Theta$ in the near wake of two bodies of revolution and calculated values of $\Theta_\infty$.

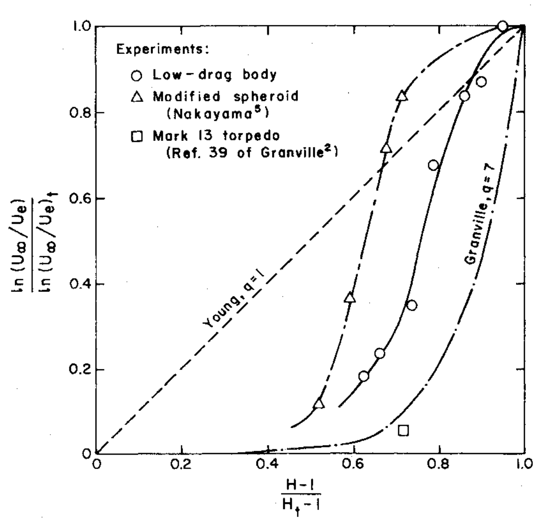


Fig. 2 Relation between $U_\infty/U_e$ and $H$ in the wakes of bodies of revolution.

an overestimate of the drag coefficient when the formula of either Young or Granville is used. On the other hand, the prediction (Fig. 1) by an iterative method[5,7,8] which takes the term $I_p$ into account is much closer to experiment.

## Conclusions

It is of interest to observe that detailed experimental data in the near wake of bodies of revolution were not available at the time Young and Granville proposed their formulas. The relative success enjoyed by these formulas over the years may be attributed largely to the fact that their application was restricted to bodies whose tail configurations were such that the boundary layer either remained thin up to the tail or separated in the actual experiments before it became thick. In either case the pressure variation would not be large enough to avoided by proper design of the tail and the boundary layer becomes thick, as in the case of the low-drag "dolphin" bodies of current interest,[6] the older drag-prediction formulas cannot be relied upon to obtain satisfactory accuracy.

As has been pointed out in earlier studies,[3,4] the variation of static pressure across the boundary layer and the near wake implies a strong interaction between the boundary layer, the wake, and the external inviscid flow. This implies that conventional boundary-layer calculation methods cannot be relied upon to predict even the boundary-layer characteristics at the tail. A drag-prediction method which accounts for the flow interaction referred to has been developed in Refs. 5, 7, and 8.

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# Water Fog Generation System for Subsonic Flow Visualization

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## Introduction

THE problems of flow visualization in a gas have in the past been related to the undesirable qualities of the injected marking smoke.[1,2] The toxicity and corrosiveness of titanium tetrachloride are well-known to anyone who has used it.[3] The vaporization of "Type 1964 Fog Juice" or the burning of a wide variety of combustibles for the purpose of producing smoke have inherent cleanliness and safety problems.[4,5] Not only is a dense smoke usually acrid and harmful to breathe, but the methods of vaporization are dangerous from the standpoint of explosion and/or fire.[4] The

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